CHAPTER 13

Study Guide and Review

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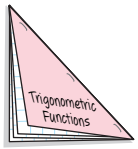
FOLDABLES Study Organizer

GET READY to Study

Be sure the following Key Concepts are noted in your Foldable.



Key Concepts

Right Triangle Trigonometry (Lesson 13-1)

- $\sin\theta = \frac{\text{opp}}{\text{hyp}}$, $\cos\theta = \frac{\text{adj}}{\text{hyp}}$, $\tan\theta = \frac{\text{opp}}{\text{adj}}$, $\csc\theta = \frac{\text{hyp}}{\text{opp}}$, $\sec\theta = \frac{\text{hyp}}{\text{adj}}$, $\cot\theta = \frac{\text{adj}}{\text{opp}}$

Angles and Angle Measure (Lesson 13-2)

- An angle in standard position has its vertex at the origin and its initial side along the positive x-axis.
- The measure of an angle is determined by the amount of rotation from the initial side to the terminal side.

Trigonometric Functions of General Angles (Lesson 13-3)

- You can find the exact values of the six trigonometric functions of θ, given the coordinates of a point $P(x, y)$ on the terminal side of the angle.

Law of Sines and Law of Cosines (Lesson 13-4 and 13-5)

- $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$
- $a^2 = b^2 + c^2 - 2bc\cos A$
- $b^2 = a^2 + c^2 - 2ac\cos B$
- $c^2 = a^2 + b^2 - 2ab\cos C$

Circular and Inverse Trigonometric Functions (Lesson 13-6 and 13-7)

- If the terminal side of an angle θ in standard position intersects the unit circle at $P(x, y)$, then $\cos\theta = x$ and $\sin\theta = y$.
- $y = \text{Sin}\, x$ if $y = \sin x$ and $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$

Key Vocabulary

- angle of depression (p. 764)
- angle of elevation (p. 764)
- arccosine function (p. 807)
- arcsine function (p. 807)
- arctangent function (p. 807)
- circular function (p. 800)
- cosecant (p. 759)
- cosine (p. 759)
- cotangent (p. 759)
- coterminal angles (p. 771)
- initial side (p. 768)
- law of cosines (p. 793)
- law of sines (p. 786)
- period (p. 801)
- periodic (p. 801)
- principal values (p. 806)
- quadrantal angles (p. 777)
- radian (p. 769)
- reference angle (p. 778)
- secant (p. 759)
- sine (p. 759)
- solve a right triangle (p. 762)
- standard position (p. 768)
- tangent (p. 759)
- terminal side (p. 768)
- trigonometric functions (p. 759)
- trigonometry (p. 759)
- unit circle (p. 769)

Vocabulary Check

State whether each sentence is *true* or *false*. If false, replace the underlined word(s) or number to make a true sentence.

1. When two angles in standard position have the same terminal side, they are called quadrantal angles. false; coterminal
2. The Law of Sines is used to solve a triangle when the measure of two angles and the measure of any side are known. true
3. Trigonometric functions can be defined by using a unit circle. true
4. For all values of θ, $\csc\theta = \frac{1}{\cos\theta}$. false; $\sec\theta$
5. A radian is the measure of an angle on the unit circle where the rays of the angle intercept an arc with length 1 unit. true
6. In a coordinate plane, the initial side of an angle is the ray that rotates about the center. false; terminal

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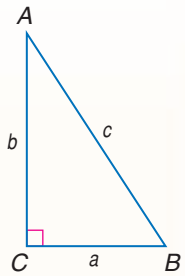
Lesson-by-Lesson Review

13-1 Right Triangle Trigonometry (pp. 759–767)

Solve $\triangle ABC$ by using the given measurements. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.



7. $c = 16, a = 7$

8. $A = 25°, c = 6$

9. $B = 45°, c = 12$

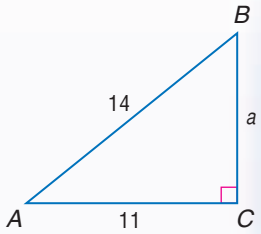
10. $B = 83°, b = \sqrt{31}$

11. $a = 9, B = 49°$

12. $\cos A = \frac{1}{4}, a = 4$

13. SKATEBOARDING A skateboarding ramp has an angle of elevation of 15.7°. Its vertical drop is 159 feet. Estimate the length of this ramp.

Example 1 Solve $\triangle ABC$ by using the given measurements. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.



Find a. $a^2 + b^2 = c^2$

$a^2 + 11^2 = 14^2$

$a = \sqrt{14^2 - 11^2}$

$a \approx 8.7$

Find A. $\cos A = \frac{11}{14}$

Use a calculator.

To the nearest degree $A \approx 38°$.

Find B. $38° + B \approx 90°$

$B \approx 52°$

Therefore, $a \approx 8.7$, $A \approx 38°$, and $B \approx 52°$.

13-2 Angles and Angle Measure (pp. 768–774)

Rewrite each degree measure in radians and each radian measure in degrees.

14. 255°

15. $-210°$

16. $\frac{7\pi}{4}$

17. -4π

Find one angle with positive measure and one angle with negative measure coterminal with each angle.

18. 205°

19. $-40°$

20. $\frac{4\pi}{3}$

21. $-\frac{7\pi}{4}$

22. BICYCLING A bicycle tire has a 12-inch radius. When riding at a speed of 18 miles per hour, determine the measure of the angle through which a point on the wheel travels every second. Round to both the nearest degree and nearest radian.

Example 2 Rewrite the degree measure in radians and the radian measure in degrees.

a. $240°$

$240° = 240°\left(\frac{\pi \text{ radians}}{180°}\right)$

$= \frac{240\pi}{180}$ radians or $\frac{4\pi}{3}$

b. $\frac{\pi}{12}$

$\frac{\pi}{12} = \left(\frac{\pi}{12} \text{ radians}\right)\left(\frac{180°}{\pi \text{ radians}}\right)$

$= \frac{180°}{12}$ or $15°$

13-3 Trigonometric Functions of General Angles (pp. 776–783)

Find the exact value of the six trigonometric functions of θ if the terminal side of θ in standard position contains the given point.

23. $P(2, 5)$ **24.** $P(15, -8)$

Find the exact value of each trigonometric function.

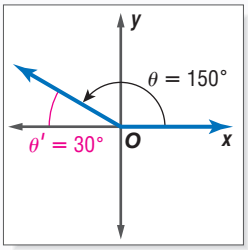
25. $\cos 3\pi$ **26.** $\tan 120°$

27. BASEBALL The formula $R = \frac{V_0^2 \sin 2\theta}{32}$ gives the distance of a baseball that is hit at an initial velocity of V_0 feet per second at an angle of θ with the ground. If the ball was hit with an initial velocity of 60 feet per second at an angle of 25°, how far was it hit?

about 86.2 ft

Example 3 **Find the exact value of $\cos 150°$.**

Because the terminal side of 150° lies in Quadrant II, the reference angle θ' is $180° - 150°$ or 30°. The cosine function is negative in Quadrant II, so $\cos 150° = -\cos 30°$ or $-\frac{\sqrt{3}}{2}$.



13-4 Law of Sines (pp. 785–792)

Determine whether each triangle has no solution, one solution, or two solutions. Then solve each triangle. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.

28. $a = 24, b = 36, A = 64°$

29. $A = 40°, b = 10, a = 8$

30. $b = 10, c = 15, C = 66°$

31. $A = 82°, a = 9, b = 12$

32. $A = 105°, a = 18, b = 14$

33. NAVIGATION Two fishing boats, A, and B, are anchored 4500 feet apart in open water. A plane flies at a constant speed in a straight path directly over the two boats, maintaining a constant altitude. At one point duing the flight, the angle of depression to A is 85°, and the angle of depression to B is 25°. Ten seconds later the plane has passed over A and spots B at a 35° angle of depression. How fast is the plane flying?

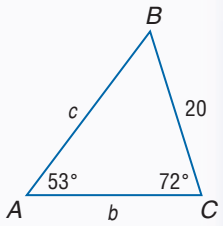
302.2 mph

Example 4 **Solve $\triangle ABC$.**

First, find the measure of the third angle.

$53° + 72° + B = 180°$

$B = 55°$



Now use the law of Sines to find b and c. Write two equations, each with one variable.

$$\frac{\sin A}{a} = \frac{\sin C}{c} \qquad \frac{\sin B}{b} = \frac{\sin A}{a}$$

$$\frac{\sin 53°}{20} = \frac{\sin 72°}{c} \qquad \frac{\sin 55°}{b} = \frac{20 \sin 53°}{20}$$

$$c = \frac{20 \sin 72°}{\sin 53°} \qquad b = \frac{20 \sin 55°}{\sin 53°}$$

$$c \approx 23.8 \qquad b \approx 20.5$$

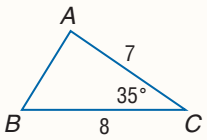
Therefore, $B = 55°$, $b \approx 20.5$, and $c \approx 23.8$.

Mixed Problem Solving
For mixed problem-solving practice, see page 938.

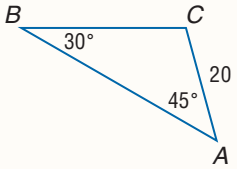
13-5 Law of Cosines (pp. 793–798)

Determine whether each triangle should be solved by beginning with the Law of Sines or Law of Cosines. Then solve each triangle. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.

34.



35.



36. $C = 65°$, $a = 4$, $b = 7$

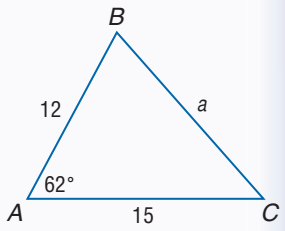
37. $A = 36°$, $a = 6$, $b = 8$

38. $b = 7.6$, $c = 14.1$, $A = 29°$

39. SURVEYING Two sides of a triangular plot of land have lengths of 320 feet and 455 feet. The measure of the angle between those sides is 54.3°. Find the perimeter of the plot.

Example 5 **$\triangle ABC$ for $A = 62°$, $b = 15$, and $c = 12$.**

You are given the measure of two sides and the included angle. Begin by drawing a diagram and using the Law of Cosines to determine a.



$a^2 = b^2 + c^2 - 2bc \cos A$

$a^2 = 15^2 + 12^2 - 2(15)(12) \cos 62°$

$a^2 \approx 200$

$a \approx 14.1$

Next, you can use the Law of Sines to find the measure of angle C.

$$\frac{\sin 62°}{14.1} \approx \frac{\sin C}{12}$$

$$\sin C \approx \frac{12 \sin 62°}{14.1} \text{ or about } 48.7°$$

The measure of the angle B is approximately $180 - (62 + 48.7)$ or $69.3°$. Therefore, $a \approx 14.1$, $C \approx 48.7°$, $B \approx 69.3°$.

CHAPTER 13

Study Guide and Review

13-6 Circular Functions (pp. 799–805)

Find the exact value of each function.

40. sin (−150°)

41. cos 300°

42. (sin 45°)(sin 225°)

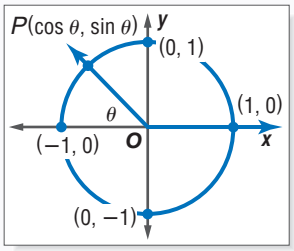
43. $\sin \frac{5\pi}{4}$

44. $(\sin 30°)^2 + (\cos 30°)^2$

45. $\frac{4 \cos 150° + 2 \sin 300°}{3}$

46. FERRIS WHEELS A Ferris wheel with a diameter of 100 feet completes 2.5 revolutions per minute. What is the period of the function that describes the height of a seat on the outside edge of the Ferris wheel as a function of time?

Example 6 Find the exact value of $\cos\left(-\frac{7\pi}{4}\right)$.



$$\cos -\frac{7\pi}{4} = \cos\left(-\frac{7\pi}{4} + 2\pi\right)$$
$$= \cos \frac{\pi}{4} \text{ or } \frac{\sqrt{2}}{2}$$

13-7 Inverse Trigonometric Functions (pp. 806–811)

Find each value. Write angle measures in radians. Round to the nearest hundredth.

47. $\text{Sin}^{-1}(-1)$

48. $\text{Tan}^{-1}\sqrt{3}$

49. $\tan\left(\text{Arcsin}\frac{3}{5}\right)$

50. $\cos(\text{Sin}^{-1} 1)$

51. FLYWHEELS The equation $y = \text{Arctan } 1$ describes the counterclockwise angle through which a flywheel rotates in 1 millisecond. Through how many degrees has the flywheel rotated after 25 milliseconds?

Example 7 Find the value of $\text{Cos}^{-1}\left[\tan\left(-\frac{\pi}{6}\right)\right]$ in radians. Round to the nearest hundredth.

KEYSTROKES: 2nd [COS^{-1}] TAN (−) 2nd [π] ÷ 6)) ENTER 2.186276035

Therefore, $\text{Cos}^{-1}\left[\tan\left(-\frac{\pi}{6}\right)\right] \approx 2.19$ radians.